

The phase connection of 4D Stringy BH to higherD black branes in canonical ensemble

Jian-Xin Lu
(with Qiang Jia and Xiao-Jun Tan)
Based on [arXiv:1603.08084](https://arxiv.org/abs/1603.08084)

University of Science & Technology of China

1st East Asia Joint Workshop on Fields& Strings, Hefei, China

May 29, 2016

Outline

- Motivation
- The phase structure of 4D Stringy BH in canonical ensemble
- Its connection to higher dimensional brane systems
- Summary

Motivation

The bosonic part of 4D low energy string effective action is given

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(\mathcal{R} - 2(\nabla\phi)^2 - e^{2a\phi} F^2 \right), \quad (1.1)$$

where ϕ the dilaton, a the dilaton coupling and F the field strength.

The general [asymptotically-flat spherically-symmetric](#) charged black hole solution is

$$\begin{aligned} ds^2 &= -\lambda^2 dt^2 + \frac{dr^2}{\lambda^2} + R^2 d\Omega_2^2, \\ F &= \frac{e^{-a\phi_0}}{(1+a^2)^{\frac{1}{2}}} \frac{(r_+ r_-)^{\frac{1}{2}}}{r^2} dr \wedge dt, \\ e^{-2a(\phi-\phi_0)} &= \left(1 - \frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}}, \end{aligned} \quad (1.2)$$

Garfinkle, Horowitz&Strominger, PRD43:3140, 1991

Motivation

In the above,

$$\begin{aligned}\lambda^2 &= \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}} \\ R &= r \left(1 - \frac{r_-}{r}\right)^{\frac{a^2}{1+a^2}}.\end{aligned}\tag{1.3}$$

In general, such a black system is not thermodynamically stable due to Hawking radiation.

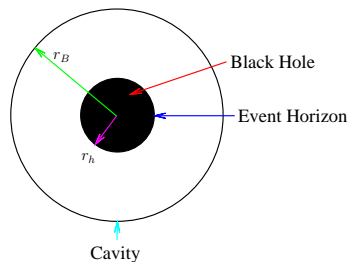
Motivation

There are two ways to deal with this instability:

- one is to add a negative cosmological constant to give rise black holes in AdS,
- the other is to place such a system in a spherical cavity outside the horizon with the temperature of the cavity wall fixed *a la* York.

Our interest here is for the latter case and considers, in particular, the charge inside the cavity also fixed, therefore defining a canonical ensemble. (York, PRD33:2092, 1986)

Motivation



Black hole (r_h) placed in a cavity (r_B) with fixed T and V .

Motivation

As is well-known, 4D stringy black holes, with the dilaton coupling

$$a^2 = \Delta - 1 = \frac{4}{N} - 1, \quad \text{with } N = 1, 2, 3, 4 \quad (1.4)$$

are related to 11 or 10D black brane and intersecting brane systems whose extremal limits are SUSY via dim. reductions.

For other values of a , the 4D stringy black holes are related to the higher D brane systems whose extremal limits are non-susy also via dim. reductions.

Lu&Pope,NPB465:127(1996);Papadopoulos&Townsend,PLB380:273(1996);Tseytlin,NPB475:149(1996);Klebanov&Tseytlin,NPB475:179(1996)...

Motivation

Given this,

it is natural to ask if there exist possible connection of the 4D stringy BH to the higher dim. brane systems in phase structures.

Thermal stability analysis

Under the setting of canonical ensemble,
the Helmholtz free energy, to leading order, is given

$$F = I_E / \beta, \quad (2.1)$$

with I_E the corresponding Euclidean action of BH or black branes
and β the inverse of preset temperature of the cavity wall.

Thermal stability analysis

The relevant quantities for the thermal stability are the following reduced ones:

$$x = \frac{R_+}{R_B} < 1, \quad q = \frac{Q}{R_B}, \quad \bar{b} = \frac{\beta}{4\pi R_B}, \quad \bar{I}_E = \frac{I_E}{4\pi R_B^2} \quad (2.2)$$

with Q the fixed charge, R_B the fixed cavity physical radius and R_+ the physical horizon radius.

Note that the reduced horizon size x is the only variable for the free energy or \bar{I}_E .

Thermal stability analysis

In general, we have

$$\frac{\partial F}{\partial x} \sim \frac{\partial \bar{I}_E}{\partial x} = c(x) [\bar{b} - b_q(x)], \quad (2.3)$$

where $c(x) > 0$ and $b_q(x) > 0$ in the allowed range of x defined as $q < x < 1$.

The free energy or \bar{I}_E takes its extremal value, from (2.3), when $b_q(\bar{x}) = \bar{b}$. Then at $x = \bar{x}$, we have

$$\left. \frac{\partial^2 \bar{I}_E}{\partial x^2} \right|_{x=\bar{x}} = -c(\bar{x}) \left. \frac{\partial b_q(x)}{\partial x} \right|_{x=\bar{x}}. \quad (2.4)$$

Thermal stability analysis

Since $c(\bar{x}) > 0$, we have a local minimum of \bar{I}_E or free energy at $x = \bar{x}$ if

$$\left. \frac{\partial b_q(x)}{\partial x} \right|_{x=\bar{x}} < 0. \quad (2.5)$$

In other words, only when the slope of $b_q(x)$ is negative at $x = \bar{x}$, the corresponding \bar{I}_E or the free energy takes its local minimal value, giving a locally (thermally) stable phase.

This is the criterion used to analyze the thermal stability of BH or black branes!

Thermal stability analysis

For 4D stringy BH (1.2) and (1.3), we have

$$b_q(x) = x\sqrt{1-x} \left(1 - \frac{q^2}{x}\right)^{\frac{1-3a^2}{2(1+a^2)}} \left(1 - \frac{q^2}{x^2}\right)^{\frac{a^2-1}{a^2+1}}, \quad (2.6)$$

and

$$c(x) = \frac{(x^2 - q^2/(1+a^2)) [x - q^2] + a^2 q^2 x(1-x)/(1+a^2)}{2 x^3 \sqrt{1-x} \left(1 - \frac{q^2}{x}\right)^{3/2}} > 0, \quad (2.7)$$

where the allowed range of x is

$$q < x < 1. \quad (2.8)$$

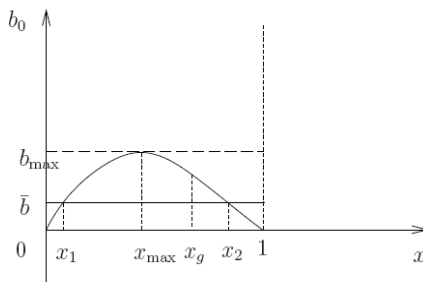
Thermal phases

0-charge case:

Now

$$b_0(x) = x\sqrt{1-x}, \quad 0 < x < 1, \quad (2.9)$$

independent of a and the behavior of $b_0(x)$ vs x is



Thermal phases

We would like to stress that this is the typical phase structure of zero-charge black systems, whether they are BH or black branes.

This is due to the two end limits: $b_0(x \rightarrow 0) \rightarrow 0$ and $b_0(x \rightarrow 1) \rightarrow 0$, both of which are finite.

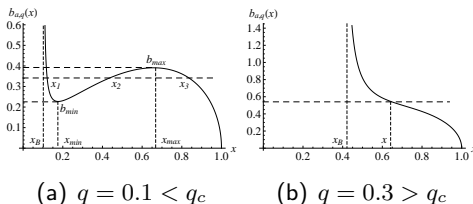
Whenever we have $b_q(x)$ taking its finite values at the two ends of allowed x , we will have a similar phase structure as the 0-charge one.

Thermal phases

For non-zero charge, we have three sub-cases to consider, depending on $0 \leq a < 1$, $a = 1$ and $a > 1$, respectively.

Case 1: $0 \leq a < 1$

Note that from (2.6) $b_q(x \rightarrow 1) \rightarrow 0$ while $b_q(x \rightarrow q) \rightarrow \infty$. This latter blowing-up behavior is a typical one of the van der Waals liquid-gas type phase structure, an illustration for $a = 0.2$ as follows:



Thermal phases

Here we also stress that $b_q(x \rightarrow 1) \rightarrow 0$ is always true and whether we have the van der Waals-Maxwell liquid-gas type phase structure depends completely on if $b_q(x)$ blows up or not at the lower end of x .

This serves the criterion for the van der Waals-Maxwell phase structure.

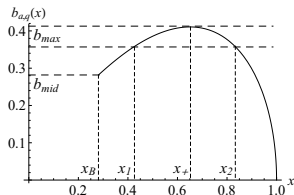
Thermal phases

Case 2: $a = 1$

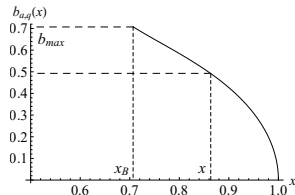
Now the $b_{1,q}(x)$ is pretty simple

$$b_{1,q}(x) = \frac{x\sqrt{1-x}}{\sqrt{1-q^2/x}}, \quad (2.10)$$

and its behavior vs x



(c) $q = 0.2$

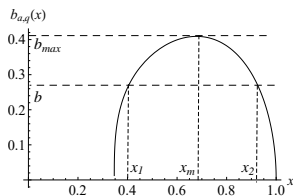


(d) $q = 0.5$

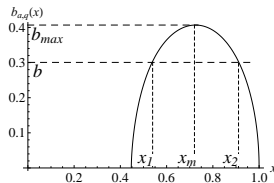
Thermal phases

Case 3: $a > 1$

For this case, we have from (2.6) $b_{a,q}(x \rightarrow q) \rightarrow 0$ and $b_{a,q}(x \rightarrow 1) \rightarrow 0$, a typical behavior of charged black p-branes with $p = D - 4$ in D dimensions (Lu, Roy, & Xiao, JHEP01:133(2011)), an illustration of this case with $q = 0.2$ for $a^2 = 2, 4$ given below



(e) $a^2 = 2$



(f) $a^2 = 4$

Thermal phases

In summary,

we have basically two kinds of phase structures, namely,
chargesless type and the van-der Waals gas-liquid type,
for all $a \geq 0$.

Dimensional reduction

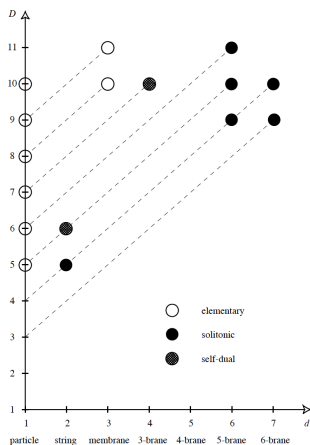
For a given p -brane in bulk D -dimensions, we have $d = p + 1$, the brane directions, and $\tilde{d} + 2 = D - 1 - p$, the directions transverse to the brane.

There are two kinds of dimensional reductions, namely,

- a direct reduction gives $p \rightarrow p$, $D \rightarrow D - 1$ or $d \rightarrow d$, $\tilde{d} \rightarrow \tilde{d} - 1$. In other words, d remains unchanged in the reduction,
- a diagonal reduction gives $p \rightarrow p - 1$, $D \rightarrow D - 1$ or $d \rightarrow d - 1$, $\tilde{d} \rightarrow \tilde{d}$. In other words, \tilde{d} remains unchanged.

10D black branes

First let us focus the 10 D simple black p-branes with $0 \leq p \leq 6$.
This gives $7 \geq \tilde{d} = 7 - p \geq 1$.



10D black branes

For these p-branes,

$$b_q(x) = \frac{1}{\tilde{d}} \frac{x^{1/\tilde{d}}(1-x)^{1/2}}{\left(1 - \frac{q^2}{x^2}\right)^{\frac{\tilde{d}-2}{2\tilde{d}}} \left(1 - \frac{q^2}{x}\right)^{\frac{1}{\tilde{d}}}}, \quad (3.1)$$

where $q < x < 1$.

We would like to stress that this $b_q(x)$ depends only on the \tilde{d} and is actually valid for all D.

The corresponding dilaton coupling

$$\alpha^2(p) = \Delta - \frac{2d\tilde{d}}{D-2} = 4 - \frac{d\tilde{d}}{4}, \quad (3.2)$$

where $\Delta = 4/N = 4$ since here $N = 1$.

10D black branes

Note again that $b_q(x \rightarrow 1) \rightarrow 0$. However, at the lower end of x ,

- $b_q(x \rightarrow q) \rightarrow \infty$ for $p \leq 4$ ($\tilde{d} \geq 3$),
- $b_q(x \rightarrow q) = \sqrt{q}/2$ for $p = 5$ ($\tilde{d} = 2$),
- $b_q(x \rightarrow q) \rightarrow 0$ for $p = 6$ ($\tilde{d} = 1$).

In other words, the $p \leq 4$ black branes have the van der Waals-Maxwell liquid-gas phase structure while $p = 5$ has the phase structure of 4D stringy BH with $a = 1$ and $p = 6$ has the phase structure of 4D stringy BH with $a > 1$.

10D black branes

Let us consider the dimensional reductions of each of these branes to 4D BH. Since the $\Delta = 4$ keeps unchange under dimensional reductions, whether direct or diagonal, so we expect that the corresponding 4D stringy BH has

$$a^2 = \Delta - 1 = 4 - 1 = 3. \quad (3.3)$$

For this $a = \sqrt{3}$, we have from (2.6) for the 4D stringy BH

$$b_q(x) = \frac{x\sqrt{1-x}\left(1 - \frac{q^2}{x^2}\right)^{1/2}}{1 - \frac{q^2}{x}}, \quad (3.4)$$

which is exactly the same as that for the 10D 6-branes ($\tilde{d} = 1$), therefore having the same phase structure.

10D black branes

This is expected since the 10D 6-branes are related to 4D stringy BH via 6-step diagonal reductions for which $\tilde{d} = 1$ remains unchanged. Therefore the function $b_q(x)$ for 10D 6-branes remains unchanged during the diagonal reductions, therefore having the same phase structure as that of the corresponding 4D stringy BH with $a = \sqrt{3}$.

Then what is the story for the other p-branes? Since the b-function for the 4D stringy BH is already fixed, therefore also its phase structure, we expect that the phase structure for other 10D p-branes cannot be preserved during their reductions to 4D. Then how to understand this? Let us consider $p = 3$ case for example.

10D black branes

Look at the brane-scan, one approach in reducing 10D 3-branes to 4D stringy BH can be done first via 3-step diagonal reductions to give stringy BH in $D = 7$, then 3 more steps via direct reductions from $D = 7$ to $D = 4$. In the first 3-steps, we have $\tilde{d} = 4$ unchanged but for each of following 3-step direct reductions, we will have $\tilde{d} \rightarrow \tilde{d} - 1$ and end up finally at $D = 4$ with $\tilde{d} = 4 - 3 = 1$, so giving precisely the b-function expected for the 4D stringy BH.

The above is true for all $p \leq 5$ and is actually independent of the path one takes.

10D black branes

The first conclusion drawn is:

Diagonal reductions preserve the phase structure while the direct reductions have the potential to change the underlying phase structure.

10D black branes

The above appears to indicate that the van der Waals-Maxwell liquid-gas type phase structure in 10D for $p \leq 4$ gets lost in the course of dimensional reductions.

This is not quite true since all possible phase structures for $p \leq 5$ -branes can be realized via adding delocalized lower dimensional branes to the 6-branes.

For example, adding delocalized $D(p-4)$ branes to Dp branes, we have (Lu, Wei & Xu, JHEP12:012(2012)) for the lower end limit of x as

$$b_{q_{p-4}, q_p}(x \rightarrow q_p) \sim \left(1 - \frac{\Delta_+}{\Delta_-}\right)^{\frac{2-\tilde{d}}{2d} - \frac{1}{2}}, \quad (3.5)$$

where we give only possible divergent factor with $\Delta_-/\Delta_+ \rightarrow 1$.

10D black branes

- So $b_{q_p-4, q_p}(x \rightarrow q_p)$ diverges if $\tilde{d} > 1$, therefore having the van der Waals-Maxwell liquid-gas type phase structure.
- For $\tilde{d} = 1$, $b_{q_2, q_6}(x \rightarrow q_6)$ is now non-vanishing finite, therefore having a phase structure of the simple charged black 5-branes.
- Follow what we did for the simple charged black p-branes via diagonal plus possible direct dimensional reductions, we end up with 4D stringy BH all with $\tilde{d} = 1$.

10D black branes

Again only for $p = 6$, corresponding to $\tilde{d} = 1$, the reductions are all diagonal ones and so the phase structure is the one of $a = 1$ 4D stringy BH.

Let us check this directly. In diagonal dimensional reductions, \tilde{d} remains unchanged. So the behavior of $b_{q_2, q_6}(x \rightarrow q_6)$ remains the same as in 10 dimensions when reduced to 4 dimensions. In comparison with the corresponding 4D stringy BH (2.6), we must have

$$\frac{a^2 - 1}{a^2 + 1} = \frac{2 - \tilde{d}}{2\tilde{d}} - \frac{1}{2}, \quad (3.6)$$

where $\tilde{d} = 1$. This equation immediately gives $a = 1$, as expected.

10D black branes

If we add delocalized D0 branes to D6, ending up with (D0, D6). Its phase structure becomes indeed the van der Waals-Maxwell liquid-gas type (Lu&Wei, JHEP04:100(2013); Lu, Ouyang&Roy, PRD90(6):066003(2014). By the same token, this gives $a = 0$ 4D stringy BH, a consistent result.

In other words, the 10D possible phase structures under the similar setting can all be captured by various configurations of D6 branes.

These further say that the 10D phase structures can also be captured by 4D stringy BH with different a .

11D intersecting branes

To lend further support to the above, we consider one more example.

As is known, the 4D stringy BH with

$$a^2 = \frac{4}{N} - 1, \quad (3.7)$$

gives $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ when $N = 4, 3, 2, 1$, respectively. The $a = 0, 1/\sqrt{3}, 1$ 4D stringy BH are actually the bound states of 4, 3, 2 of the $a = \sqrt{3}$ 4D stringy BH, respectively (Rahmfeld, PLB 372:198(1996)).

They correspond to 10 or 11D $N = 4, 3, 2$ intersecting branes, too.

11D intersecting branes

For example, the 11D $2\perp 2\perp 5\perp 5$ intersecting brane system with $\tilde{d} = 1$ is related to the $a = 0$ 4D stringy BH via diagonal reductions. When setting any of the brane charge vanish in the intersecting brane system, the resulting system is related to the $a = 1/\sqrt{3}$ and so on.

One can show that the b-function for the intersecting brane system with N intersecting branes is

$$b_q(x) \sim \left(1 - \frac{q^2}{x^2}\right)^{\frac{2-N}{2}} = \left(1 - \frac{q^2}{x^2}\right)^{\frac{a^2-1}{a^2+1}}, \quad (3.8)$$

where we have used (3.7). This is precisely the b-function for 4D stringy BH with $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ when $N = 4, 3, 2, 1$, respectively.

11D intersecting branes

So once again, the 4D stringy BH with $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ capture the phase structures of the corresponding 11D intersecting brane system, respectively.

Summary and Conclusion

The above study indicates 4D stringy BH with a general dilaton coupling $a \geq 0$ should give a general description of higher dimensional brane systems, simple or intersecting/known or unknown, in phase structure under similar setting.

This further implies that all such systems can either have the van der Waals-Maxwell liquid-gas type or the chargeless type phase structure, nothing more.

THANK YOU!